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FAST ASYMPTOTIC FORM OF THE RESISTANCE OF BODIES IN A WAVEGUIDE LAYER OF NON-UNIFORM FLUIDS†

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The asymptotic dependence of the wave resistance of bodies moving horizontally at a high speed in a waveguide with an arbitrary stratified fluid is analysed. For a waveguide of finite depth, it is established that the resistance is inversely proportional to the square of the velocity and directly proportional to the square of the volume, for small bodies. The general results are refined for uniform stratification and a pronounced transition layer.

WHEN bodies move in a density-stratified fluid internal waves are excited and perturbations propagate inside the fluid. By virtue of this fact, even if viscous resistance is neglected (in the ideal-fluid approximation) the body will experience wave resistance. It is convenient, when calculating this, to replace the boundary-value problem of the flow around the body by the problem of the motion of mass or force sources, which are equivalent to the body in their hydrodynamic effect on the fluid. These might be mass dipole sources, distributed over the surface of the submerged body and found from the solution of the boundary integral equations, for example. The use of model distributions of sources is especially helpful because it enables a number of general conclusions to be drawn without having to solve the quite time-consuming problem of the specific form of the source distributions.

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The asymptotic character of the dependence of wave resistance on the velocity of rapidly moving bodies can be established for a fairly arbitrary stratified liquid, without going into much detail on the equivalent source distributions. Only the most general assumptions need to be made concerning the latter: there is total compensation of sources and sinks in the direction of motion, the characteristic scales of the bodies and corresponding source distributions are comparable, and the bodies are small compared with the depth of the waveguide.

1. THE WAVE RESISTANCE FOR A HORIZONTAL WAVEGUIDE WITH ARBITRARY STRATIFICATION

In the linear approximation, small perturbations of the hydrodynamic characteristics from uniformly horizontally moving mass sources $m\ (\mathbf{r}-\mathbf{v}_0t, z)$, to which we shall confine ourselves below, are proportional to those sources. From the equations of the mass and momentum balance of a non-uniform, density-stratified fluid situated in a gravity field, for the vertical component of the perturbation velocity w it follows that

$$\left\{\left(\frac{\partial^2}{\partial t^2}+N^2(z)\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\frac{\partial^4}{\partial t^2\partial z^2}\right\}w=\frac{\partial^3 m}{\partial t^2\partial z}$$
$$N^2(z)\equiv gd\ln\rho/dz$$

Here, the conventional Boussinesq approximation has been used, according to which the influence of changes in density on the inertia is neglected, $N(z)$ is the buoyancy frequency, the square of which is proportional to the initial density gradient, which depends only on the vertical coordinate z and $\mathbf{r}=(x, y)$ is the horizontal coordinate vector.

It is convenient to represent the solution of this differential equation as a Fourier integral expansion with respect to the horizontal variables and time, and a series in the complete system of eigenfunctions which depend on the vertical coordinate.

$$w=\sum_n\int d^2k\, d\omega w_n(\mathbf{k}, z)\,\Phi_n(\mathbf{k},\omega)\,e^{i\mathbf{kr}-i\omega t} \tag{1.1}$$
$$\Phi_n=\frac{1}{4\pi^2}\frac{\omega^2c_n^2k^2}{(\omega+i\varepsilon)^2-c_n^2k^2}\delta(\omega-\mathbf{kv}_0)\int_{h_1}^{h_2}dz\,m(\mathbf{k},z)\frac{\partial w_n(k,z)}{\partial z}$$

Here $\mathbf{k}=(k_x, k_y)$ is the wave vector, $k=|\mathbf{k}|$ and $m(\mathbf{k}, z)$ is the Fourier component of mass source $m(\mathbf{r}-\mathbf{v}_0t, z)$. The appearance of the delta-function $\delta(\omega-\mathbf{kv}_0)$ in the expansion is due to the dependence of the latter on the difference argument $\mathbf{r}-\mathbf{v}_0t$. The notation $+i\varepsilon$ indicates a small shift of the poles from the real frequency axis.

A spatially non-uniform stratification (along the vertical) has waveguide properties. The corresponding eigenfunctions $w_n(k, z)$ and eigenvalues $c_n=c_n(k)$ are determined by the solution of the spectral problem

$$(\partial^2/\partial z^2-k^2+c_n^{-2}N^2(z))\,w_n=0,\qquad w_n|_{z=h_1}=w|_{z=h_2}=0 \tag{1.2}$$

The wave mode spectrum is discrete if the waveguide is formed by rigid horizontal boundaries situated at finite depths $z=h_1$ and $z=h_2$, or by a stratification that disappears at great depths ($N^2\to 0$ as $z\to h_{1,2}=\pm\infty$) in the case of an infinite fluid [1]. Only cases of this kind will be considered below.

Perturbations of the pressure p can be expressed in terms of the vertical velocity component and written in a form similar to (1.1)

$$p=\sum_n\int d^2k\,d\omega P_n(\omega,\mathbf{k},z)\,e^{i\mathbf{kr}-i\omega t}$$
$$P_n=\frac{i\omega\rho}{k^2}\left\{\frac{\partial w_n(k,z)}{\partial z}\Phi_n-\frac{1}{(2\pi)^2}m(\mathbf{k},z)\,\delta(\omega-\mathbf{kv}_0)\right\} \tag{1.3}$$

In the linear problem energy characteristics, like the wave resistance, which are quadratic with respect to perturbations, become quadratic forms of the source distributions. For a uniformly moving distribution of mass sources, taking (1.1) and (1.3) into account, the wave resistance R can be represented by the following sum of the contributions of the individual wave modes [2]:

$$R = \frac{1}{v_0}\int d^2r \int_{h_1}^{h_2} dz\, pm = \sum_n R_n$$

$$R_n = \frac{\rho}{2\pi v_0}\int d^2r\, d^2r' \int_{h_1}^{h_2} dz\, dz' S_n(\mathbf{r}-\mathbf{r}'; z, z')\, m(\mathbf{r}, z)\, m(\mathbf{r}', z') \tag{1.4}$$

$$S_n = \int_0^\infty dk \frac{\theta(v_0 - c_n)}{\sqrt{v_0^2 - c_n^2}} k^2 c_n^4 \frac{\partial w_n}{\partial z}\frac{\partial w_n}{\partial z'} \cos\frac{k(x-x')c_n}{v_0} \times$$
$$\times \cos\left(\frac{k(y-y')}{v_0}\sqrt{v_0^2 - c_n^2}\right)$$

Here $\theta(v_0 - c_n)$ is the Heaviside unit function.

The important properties of the spectral problem for internal waves (1.2) are the monotonic decrease in the phase velocities of the waves $c_n(k)$ as the wave number k and mode number n increase, the limitation on the possible frequencies $\omega_n(k) = kc_n(k) \leqslant N_{\max}$ and the boundedness of the phase velocities of all waves in the special case of a waveguide of finite depth [1, 2]. Thanks to these properties, the general expression for the contribution of the nth mode to the wave resistance R_n is easily simplified for high source velocities.

For velocities of the modelled body exceeding the phase velocities of waves of the given mode $v_0 > c_{n0} \equiv \lim_{k\to 0} c_n(k) \geqslant c_n(k)$ (which is always possible in a waveguide of finite depth), the Heaviside function does not place any restrictions on the range of integration with respect to the wave numbers and can be omitted. For even higher velocities ($v_0 \gg c_{n0}$) $\sqrt{(v_0^2 - c_{n0}^2)}$ can be replaced by v_0.

When replacing the body by mass sources, it is natural to require total compensation of all sources and sinks. We shall assume that this applies in the direction of motion (assuming that the body has a certain symmetry), which we shall take to coincide with the direction of the x axis

$$\int dx\, m(\mathbf{r}, z) = 0, \quad D = \int_{h_1}^{h_2} dz\, D(z) = \int_{h_1}^{h_2} dz \int d^2r\, x\, m(\mathbf{r}, z) \neq 0 \tag{1.5}$$

Then $S_n|_{x=x'}$ is not reflected in the value of the resistance according to (1.4), and instead of $\cos(kc_n(x-x')/v_0)$ we can put $\cos(kc_n(x-x')/v_0) - 1$. Assuming, in addition to $v_0 \gg c_{n0}$, that the body is relatively small ($l_x \ll v_0/N_{\max}$, $l_y \ll h_2 - h_1$), the expression for the contribution of the nth mode to the wave resistance can be simplified even more:

$$R_n = \frac{\rho}{2\pi v_0^4}\int_{h_1}^{h_2} dz\, dz' A_n(z, z')\, D(z)\, D(z')$$
$$A_n \approx \int_0^\infty dk\, k^4 c_n^6 \frac{\partial w_n(k, z)}{\partial z}\frac{\partial w_n(k, z')}{\partial z'} \tag{1.6}$$

The function $A_n(z, z')$ is determined exclusively by the stratification and the type of waveguide, rather than the parameters (and the velocity, in particular) of the sources. Apart from the multiplier v_0^{-4}, the value of the contribution to the resistance R_n may depend implicitly on the velocity in terms of the vertical density of dipole moments $D(z)$. However, at high velocities, in the framework of the linear description of small perturbations, we would expect this dependence to be similar to a linear dependence $D(z) \approx v_0 \Delta(z)$, which applies to a uniform fluid. Finally, the asymptotic dependence of the contribution to the wave resistance of the nth mode on the velocity turns out to be a decreasing power function:

$$R_n \sim v_0^{-2}, \quad v_0 \gg c_{n0},\ N_{\max} l \tag{1.7}$$

For a waveguide of finite depth, the velocities of all internal waves are finite, so that $v_0 \gg c_{10} \geq c_n(k)$, and for all modes the estimate (1.7) can be used. So, for the total wave resistance, assuming that the series can be summed over the modes, we have the asymptotic estimate

$$R \sim F^{-2}, \quad F \equiv v_0/(N_{\max} l) \gg 1 \tag{1.8}$$

It is a different situation in the case of a fluid which is infinite in a vertical direction. Apart from wave modes ($n \geq 1$) with limited wave velocities, there is a lowest "zero" mode, for which the eigenvalue increases without limit as the wave number decreases [1]

$$c_0^2(k)|_{k\to 0} \approx \frac{\gamma g}{k} + O(1), \quad \gamma \equiv \frac{\rho(+\infty) - \rho(-\infty)}{\rho(+\infty) + \rho(-\infty)} \tag{1.9}$$

Thus, estimates (1.6) and (1.7) can only be used for higher modes with $n \geq 1$. For the zero mode, the inequality $v_0 \gg c_0(k)$ cannot be satisfied for all wave numbers and the range of integration in (1.4) with respect to the wave numbers for the contribution to the wave resistance of the zero mode will always depend on the value of the velocity. Thus, the resulting asymptotic dependence on the velocity will be more complicated. The only simplifications in general form that can be made of the contribution of the zero mode are not very significant:

$$\begin{gathered} R_0 = \frac{\rho}{2\pi v_0^4} \int_{l_1}^{h_2} dz\, dz' A_0(z, z') D(z) D(z') \\ A_0 \approx v_0 \int_0^\infty dk \frac{\theta(v_0 - c_0)}{\sqrt{v_0^2 - c_0^2}} k^4 c_0^6 \frac{\partial w_0(k, z)}{oz} \frac{\partial w_0(k, z')}{\partial z'} \end{gathered} \tag{1.10}$$

Below, we refine the general argument for the specific case of a uniform stratification, and a stratification with a sharp maximum of the buoyancy frequency.

2. A UNIFORMLY STRATIFIED FLUID IN A WAVEGUIDE OF FINITE DEPTH

In the case of a uniformly stratified layer of fluid (with constant buoyancy frequency N) of depth h, enclosed between rigid boundaries $z = -h_1 = 0$ and $z = h_2 = h$, the spectral problem (1.2) has the simple solution

$$c_n^2 = \frac{N^2 h^2}{k^2 h^2 + \pi^2 n^2}, \quad w_n^2 = \frac{2h}{N^2 k^2 h^2} \sin^2 \frac{\pi n z}{h}, \quad n = 1, 2, \ldots$$

Here the simple conditions $c_n^2 < c_{n0}^2 = N^2 h^2/(\pi^2 n^2)$ apply to the velocities of all internal waves. It becomes clear that the condition of large velocities $v_0 \gg c_{n0}$ implies a large value for the Froude number of the waveguide $v_0/(Nh)$. For relatively small bodies ($l \ll h$) this is even more so for the Froude number $F \equiv v_0/(Nl) \gg 1$, and formula (1.6) simplifies to the form

$$R_n \approx \frac{\rho N^4}{16\pi v_0^4 n} \left(\int_0^h dz D(z) \cos \frac{\pi n z}{h} \right)^2 \tag{2.1}$$

Assuming that the vertical dimensions of the body are so small that $l_z \ll h/n$, and the horizontal of its motion z_0 is far from the horizontal of the extremum of the eigenfunction ($\cos(\pi z_0 n/h) \sim 1$), we obtain an even simpler estimate of the contribution of the nth mode to the wave resistance, namely,

$$R_n \approx \frac{\rho N^4 D^2}{16\pi v_0^4 n} \cos^2 \frac{\pi n z_0}{h} \tag{2.2}$$

Thus, the generation of the lowest wave modes at high velocities has a simple dipole character, except for "extremal" horizontals. In the case of a point dipole, this result is obtained with the sole condition $v_0 \gg Nh/n$ (without that condition, for a dipole, R_n can be expressed in terms of elliptic integrals).

If the horizontal of the body coincides with that of the extremum of the waveguide mode, it follows from (2.1) that the large-velocity contribution of the latter to the wave resistance will be determined by a certain quadrupole moment of the source distribution

$$R_n \approx \frac{\rho\pi N^4 n}{16 v_0^4 h^2} Q^2, \quad Q \equiv \int d^2r \int_{l_1}^{l_2} dz z (z - z_0)\, m(\mathbf{r}, z)$$

The contributions of all the modes can be summed using the approximate formula (2.1). For large velocities ($v_0 \gg c_{10} = Nh/\pi$) and relatively small bodies ($l \ll h$) the total wave resistance is

$$R \approx \frac{\rho N^4}{32\pi v_0^4} \int_{l_1}^{l_2} dz\, dz' D(z)\, D(z') \ln \left| \frac{h}{2\pi (z - z') \sin(\pi z_0/h)} \right|$$

It is clear from this, in particular, that the sum of the resistances over all modes is logarithmically large for modelling sources localized along the vertical. The reason for this lies in the overstatement of the role of the highest modes when bodies are modelled in this way. The formula obtained as a result also supports the general asymptotic estimate (1.8).

3. TRANSITION ZONE OF DENSITY CHANGE

In an infinite fluid, if the stratification maximum $N(z) = N_m(\operatorname{ch} z/h)^{-1}$ is pronounced, the eigenvalues have the simple form

$$c_n^2 = \frac{N_m^2 h^2}{(kh + n)(kh + n + 1)}, \quad n = 0, 1, 2, \ldots \tag{3.1}$$

and the normalized eigenfunctions can be expressed in terms of a polynomial-type hypergeometric function, i.e. Jacobi polynomials $P_n^{(\alpha,\beta)}(\xi)$:

$$\begin{aligned} w_n(k, z) &= C_n \left(\operatorname{ch} \frac{z}{h}\right)^{-kh} P_n^{(kh,\, kh)} \left(\operatorname{th} \frac{z}{h}\right) \\ C_n^2 &= \frac{(2n + 1 + 2kh)\, \Gamma(n+1)\, \Gamma(n + 1 + 2kh)}{2k^2 h N_m^2 4^k\, \Gamma^2(n + 1 + kh)} \end{aligned} \tag{3.2}$$

Owing to the lack of any strong restrictions along the vertical in this example, the long internal waves of the zero mode can be as fast as desired:

$$c_0^2 = \frac{N_m^2 h}{k(1 + kh)}, \quad w_0^2 = \frac{|2h\Gamma(2 + 2kh)|^{1/2}}{2khN_m 2^{kh}\Gamma(1 + kh)} \left(\operatorname{ch} \frac{z}{h}\right)^{-kh}$$

and so the need for the inequality $v_0 \geqslant c_0(k)$ to be satisfied [see (1.4)] has excluded the generation of long internal waves of the zero mode with wave numbers which do not satisfy the inequality

$$kh(1 + kh) \geqslant N_m^2 h^2 / v_0^2 \tag{3.3}$$

However, at high velocities ($v_0^2 \gg N_m^2 h^2$) of a much more deeply submerged body ($z_0 \gg h, l_z$), the major contribution to the integral for S_0 is given by finite, but small wave numbers ($kh \ll 1$). For small bodies (l_x, $l_y^2 z_0^{-1} \ll v_0^2/(N^2 h)$) this gives the simpler results

$$R_0 \approx \frac{\rho\mu^4}{(32\pi\mu z_0)^{1/2}} \left(\int dz D(z)\, e^{-\mu z}\right)^2, \quad \mu \equiv \frac{N_m^2 h}{v_0^2} \tag{3.4}$$

$$R_0 \approx \frac{\rho D^2 N_m^7 h^{7/2}}{v_0^7 (32\pi z_0)^{1/2}} \exp\left(-\frac{N_m^2 h z_0}{v_0^2}\right) \tag{3.5}$$

the one transforming to the other for $l_z \ll \mu^{-1}$. The fact that the asymptotic dependence on velocity here is exponential is due to the aforementioned restriction on the wave numbers of generated waves (3.3).

For sufficiently high velocities ($v_0^2 > c_{10}^2 = N_m^2 h^2/2$) there will be no restrictions on the wave numbers of the generation of the remaining modes. It is then easy to establish asymptotic dependences on the velocity of the type (1.6) and (1.7). On the assumption that the body is deep-submerged and has small sizes ($l_x \lesssim v_0/N_m$, $l_y \ll z_0$) the dependences on the other parameters are also easy to estimate (because infinitely small wave numbers make a decisive contribution)

$$R_n \approx \frac{3\rho N_m^4 D^2}{64\pi v_0^4}\left(1 - \frac{N_m^2 h^2}{n(n+1)v_0^2}\right)^{-1/2} \frac{2n+1}{n^3(n+1)^3}\left(\frac{h}{z_0}\right)^5, \quad n \geqslant 1 \tag{3.6}$$

Here, unlike the previous case, the decrease with depth is of power-function form. The contributions of the highest modes to the wave resistance decrease more rapidly as the mode number increases than in the case of uniform stratification.

As the thickness of the transition stratified layer decreases ($h \to 0$), provided that the overall density gradient is preserved ($N_m^2 h = \gamma g = \text{const}$), the density gradient increases and the contributions of all, apart from the zero mode, decrease rapidly ($R_n \sim h^3$).

We will now consider a limiting case.

4. THE INTERFACE BETWEEN FLUIDS WITH DIFFERENT DENSITIES

In the case of degenerate stratification with one surface of density change $z = 0$ separating two uniform fluids with densities $\rho(+\infty)$ and $\rho(-\infty)$, only a surface zero mode with phase wave velocities $c = (\gamma g/k)^{1/2}$ is possible [compare with (1.9)].

The formula for the wave resistance of the distribution of sources moving below [plus sign and $\rho = \rho(+\infty)$] or above [minus sign and $\rho = \rho(-\infty)$] the surface of the discontinuity can then be written in the general form [2]

$$R = \int d^2r\, d^2r' \int_0^\infty dz\, dz' S(\mathbf{r} - \mathbf{r}'; z, z')\, m(\mathbf{r}, z)\, m(\mathbf{r}', z')$$

$$S(\mathbf{r}; z, 0) = \frac{\rho(1 \pm \gamma)\nu^2}{2\pi} \int_0^\infty d\varphi\, \mathrm{ch}^2\varphi \exp(-\nu |z| \mathrm{ch}^2\varphi) \times$$
$$\times \cos(\nu x \,\mathrm{ch}\,\varphi) \cos(\nu y \,\mathrm{sh}\,\varphi\, \mathrm{ch}\,\varphi)$$

For relatively small source distributions of dipole type (1.5) which are far from the surface of the density discontinuity ($l \ll \nu^{-1} \equiv v_0^2/(\gamma g) \ll |z_0|$), we obtain the fast asymptotic form

$$R \approx \rho(1 \pm \gamma)\nu^4 (32\pi\nu |z_0|)^{-1/2} \exp(-2\nu |z_0|)$$

which approaches (3.5) as $\mu \to \nu$, $\gamma \ll 1$, that is, in the limit of a weak density discontinuity $h \to 0$, $N_m^2 h = \gamma g = \text{const}$ (limiting transition with fixed density gradient).

An asymptotic simplification of the formula for the resistance of a body which lies deep below the discontinuity layer ($\gamma g z_0 \gg v_0^2$) can be made by using a less rigorous condition on its size. If it is assumed that $\gamma g l \lesssim v_0^2$, then the results will be slightly more complicated, owing to some allowance being made for interference between waves generated from different parts of the body. For a symmetrical extended body we have

$$R \approx \rho(1 + \gamma)\nu^2 (32\pi\nu z_0)^{-1/2} M^2 \exp(-2\nu z_0)$$

$$M = \int d^2r\, dz\, m(\mathbf{r}, z) \sin(\nu x) \exp(-\nu(z - z_0))$$

The situation changes fundamentally when allowance is made for at least one boundary situated at a finite depth. The phase velocities of waves even of the zero mode then become bounded (the maximum velocity is proportional to the square root of the depth). For example, for a two-layer fluid with a rigid lid above ($z = -H$) or below ($z = +H$), this is evident from the formula

$$c^2=\gamma gk^{-1}(1-\exp(-2kH))/(1\mp\gamma \exp(-2kH))$$

Ultimately, the need for special consideration of the zero mode can be dispensed with and the fast asymptotic form of the contributions to the resistance of all modes can be found in the same way.

5. CONCLUSION

Thus, without solving the problem of flow around specific bodies of a stratified liquid at high velocities (large Froude numbers), we have managed fairly easily to find the asymptotic form of the dependence of the wave resistance on the velocity (the dependence of a quadratic functional of the solution on the parameter).

At a finite depth in the fluid, the contribution of each mode to the resistance falls off quadratically as the velocity increases ($R_n \sim v_0^{-2}$) at high velocities, and this is true for any stratification. However, the way in which this contribution decreases as the mode number increases is sensitive to the type of stratification and the relative dimensions of the body.

For a uniformly stratified liquid in a waveguide of finite depth and for source distributions which are localized in a vertical direction, the decrease in the contributions as the mode number increases is so slow ($R_n \sim n^{-1}$) that the total resistance becomes anomalously large (the series in the modes is logarithmically divergent). This corresponds directly to the infinite wave resistance of vertically localized sources moving in an infinite uniformly stratified fluid [3]. In both cases, the paradox indicates that the contribution of transverse waves is exaggerated in the modelling of three-dimensional bodies by localized sources.

The dependence on the mode number becomes stronger when the stratification is less uniform. From the arguments given above, it is clear that when a small body is a long way from the range of maximum stratification, the contributions of the highest modes of the wave resistance turn out to be negligibly small for any sources [see (3.6)].

Finally, a feature of the fast asymptotic form that should be emphasized is that the wave resistance of small bodies depends only on a general parameter, such as the total dipole moment of the model sources, proportional to the volume of the body. The only exceptions are cases where the motion is at the levels of extrema of the eigenfunctions of the waveguide.

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